

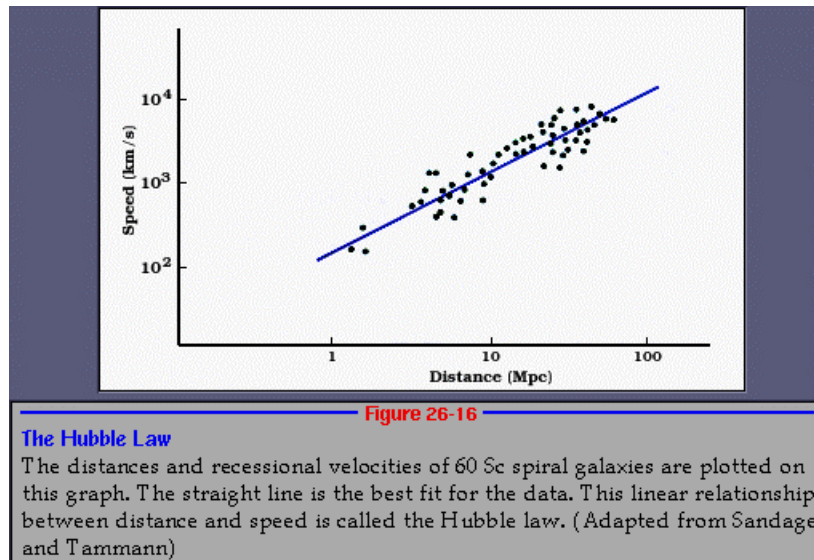
The Age of the Universe

(A Sample Problem)

1) Assume that the Hubble constant, H_0 is 75km/sec/Mpc, calculate the age of the Universe if (a) the universe is empty (i.e. no mass), and if (b) the universe has a critical density. (c) What is the age of the universe if it is denser than the critical density? (d) What's the age if it is less dense?

2) The oldest globular clusters are $\sim 13 \pm 1$ billion years old. Can either (a) or (b) be ruled out? Since we know that there is mass in the universe (we exist!), suggest an alternative cosmological.

To solve this problem let's first understand the relation between the Hubble constant and the age of the universe. Hubble was the first to notice that galaxies are moving away. In fact, galaxies that are more distant are moving away faster. Hubble's famous plot (see below) of galactic distances versus their recessional velocities demonstrates this.



Hubble found that this relationship (based on observations) is linear. The relationship between the distance and the velocity is thus the slope of the line.

$$\text{slope} = H_0 = \frac{v}{d}$$

This relationship is known as Hubble's law. It implies that the universe is expanding and that farther galaxies are moving away faster.

Since the velocity is measured in km/sec, and the distances to the galaxies in Mpc, this means that the units of the Hubble constant are “km per second per Mpc”.

$$H_o = \frac{v \left[\frac{km}{sec} \right]}{d [Mpc]}, \text{ or km/sec/Mpc.}$$

Hubble’s original measurement of the Hubble constant was 530 km/sec/Mpc. But since then it has been revised and updated, and today’s values are all much lower. Although the exact value for the Hubble constant is not certain, it lies somewhere between 50 to 100 km/sec/Mpc. The most modern values are ~80 km/sec/Mpc.

The implications of the Hubble constant are important. It says that more distant galaxies move away faster. This means that sometime in the past, everything was closer, and even at one single point. It implies an explosion of the universe, or a “Big Bang”. When did this happen? Since we know the relationship between distance and velocity, we can calculate that.

The velocity, v , is the distance traveled per time (8th grade algebra)

$$v = \frac{d}{t}$$

solving this for time gives: $t = \frac{d}{v}$

Now the Hubble constant is $H_o = \frac{v}{d}$

Solving this for velocity gives $v = d \times H_o$.

Combining these two equations and substituting for the velocity, we can obtain the time since the Big Bang:

$$t = \frac{d}{v} = \frac{d}{d \times H_o} = \frac{1}{H_o}$$

This means that the age of the universe is given by the inverse of the Hubble constant!!

If H_o turns out to be 75 km/sec/Mpc, then the age for a universe is:

$$t_{empty} = \frac{1}{H_o} = \frac{1}{75 \left[\frac{\left(\frac{km}{sec} \right)}{Mpc} \right]}$$

To solve this we first need to change the units from Mpc to km.

$$1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$$

$$1 \text{ Mpc} = 10^6 \text{ pc} = 10^6 \times 3.09 \times 10^{16} \text{ m} = 3.09 \times 10^{22} \text{ m} = 3.09 \times 10^{19} \text{ km}$$

And the age of the universe is:

$$t_{\text{empty}} = \frac{1}{H_o} = \frac{1}{75 \left[\frac{\left(\frac{\text{km}}{\text{sec}} \right)}{\text{Mpc}} \right]} = \frac{1}{75 \left[\frac{\left(\frac{\text{km}}{\text{sec}} \right)}{3.09 \times 10^{19} \text{ km}} \right]} = \frac{1}{2.43 \times 10^{-18} \frac{1}{\text{sec}}} = 4.12 \times 10^{17} \text{ sec}$$

So the age of the universe for a Hubble constant of 75 km/sec/Mpc is 4.12×10^{17} seconds. To most people this number is meaningless, therefore, let's convert seconds to years. One year has 365 days, each day has 24 hours, each hour 60 minutes and each minute has 60 seconds.

$$1 \text{ yr} = 365 \text{ days/year} \times 24 \text{ hours/day} \times 60 \text{ min/hour} \times 60 \text{ sec/min} = 3.16 \times 10^7 \text{ sec}$$

Converting seconds to years we get:

$$t_{\text{empty}} = 4.12 \times 10^{17} \text{ sec} = 4.12 \times 10^{17} \text{ sec} \times \frac{1 \text{ yr}}{3.16 \times 10^7 \text{ sec}} = 1.308 \times 10^{10} \text{ yrs}$$

$$\Rightarrow t_{\text{empty}} = 1.3 \times 10^{10} \text{ yrs}$$

However, we know that there is mass in the universe, and masses gravitate, i.e., they gravitationally attract each other. This means that the expansion is slowing down. (This, in turn, implies that the Hubble constant was larger in the past, which in turn implies that it is not really a constant!). Since the expansion was more rapid in the past than it is today, it means that the universe cannot be as old as it would be if there is no mass. Therefore we refer to the above the universe as the “empty universe”. (And we calculated the age of the empty universe, t_{empty} .)

If, however, there is a lot of mass in the universe, one can argue that the expansion may be slowed down enough, so that the expansion could be reversed. If the universe has exactly the right amount of mass, it will neither stop expanding, but not re-contract either. This model is referred to as the “critical universe”.

The age of the critical universe can be calculated (using calculus), and it turns out that

$$t_{\text{critical}} = \frac{2}{3} \frac{1}{H_o}$$

Overall there are 3 universe models: one that has little mass, one that has the critical amount of mass, and one that has more than the critical amount of mass. These are referred to as the “open”, “critical” and “closed” universe models. The ages of these three models are:

$$\begin{aligned} 1) \text{ open: } & \frac{1}{H_o} \geq t_o > \frac{2}{3} \frac{1}{H_o} \\ 2) \text{ critical: } & t_o = \frac{2}{3} \frac{1}{H_o} \\ 3) \text{ closed: } & t_o < \frac{2}{3} \frac{1}{H_o} \end{aligned}$$

For an H_o of 75 km/sec/Mpc, the age for a universe with no deceleration (i.e., not no mass – referred to as the “empty” universe) is (as before):

$$t_{\text{empty}} = \frac{1}{H_o} = \frac{1}{75 \left[\frac{\text{km}}{\frac{\text{sec}}{\text{Mpc}}} \right]} = 1.308 \times 10^{10} \text{ years}$$

and the age for a critical universe:

$$t_{\text{critical}} = \frac{2}{3} \frac{1}{H_o} = \frac{2}{3} \times 1.308 \times 10^{10} \text{ yrs} = 8.7 \times 10^9 \text{ yrs}$$

So the ranges of ages for any universe with a Hubble constant of 75 km/sec/Mpc are:

$$\begin{aligned} 1) \text{ open: } & 13 \times 10^9 \text{ yrs} \geq t_o > 8.7 \times 10^9 \text{ yrs} \\ 2) \text{ critical: } & t_o = 8.7 \times 10^9 \text{ yrs} \\ 3) \text{ closed: } & t_o < 8.7 \times 10^9 \text{ yrs} \end{aligned}$$

The ages of the globular clusters have been measured to be 13 ± 1 billion years. Since globular clusters exist, they must have formed after the Big Bang, which basically means that critical and open universe models are not possible. If the globular clusters are a little less than 13 billions years old (there is a measurement uncertainty in the globular cluster ages) and if there is very little mass in the universe, than it could possible that they have formed immediately after the Big Bang. Thus the above scenario with a Hubble constant of 75 km/sec/Mpc is only possible if the universe is open.

But this depends on the total amount of mass in the universe. At present we have detected only one fraction (1/3) of the mass that is required to close the universe. So the proposed model is

barely possible... However, we are uncertain about the total amount of dark matter in the universe, and if it turns out that there is more dark matter than we have presently detected, then we start encountering real problems.

If it turns out that the Hubble constant is 75 km/sec/Mpc or larger and globular clusters are 13 billions years old, then something must be wrong with the theory. Either a totally new theory needs to be invented, or the old one needs to be adjusted somewhat.

Einstein, when he originally came up with his General Theory of Relativity, found that there is a contraction of the universe. At that time, he believed in a static universe. To keep the universe from contracting he inserted a term into his equations (the “cosmological constant”) in order to counter-balance this contraction. This term can be interpreted as an “anti-gravity” term. Later, when Hubble discovered the expansion of the universe, Einstein revised his theory and deleted the cosmological constant. He referred to the cosmological constant as “the biggest blunder in his life”.

So, if we need a different universe model, the most convenient thing to do is to re-include the cosmological constant. This would mean that there is a repulsive (gravity opposing) force in the universe. However this cosmological constant will have to be small, because the universe IS still (or again) expanding. In fact, during the very early universe, when the expansion was more rapid than it is today, the cosmological constant was relatively unimportant. But as the universe slowed down, this term will have become more and more important, and, since it is a repulsive force, will eventually make the universe expand for ever and ever. In fact, in such a model the expansion would accelerate...

Now, if we are at a time where the universe is accelerating again – recall, everything is moving away from us – then it is conceivable that the age of the universe is older than in the open universe model. This is illustrated in the plot of your one-page class handout.

So in summary the ages of the universe for all 5 different models are:

- | | |
|------------------------------------|--|
| 1) non-zero cosmological constant: | $t_o > \frac{1}{H_o}$ |
| 2) no mass (empty): | $t_o = \frac{1}{H_o}$ |
| 3) open: | $\frac{1}{H_o} \geq t_o > \frac{2}{3} \frac{1}{H_o}$ |
| 4) critical: | $t_o = \frac{2}{3} \frac{1}{H_o}$ |
| 5) closed: | $t_o < \frac{2}{3} \frac{1}{H_o}$ |